Developing a Numerical Platform for Simulation-Based Exploration of Behavioral Economic Dynamics

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***Summary**: Behavioral Economics has a rich tradition of empirical studies involving the effects of personal decision biases involving trade-offs between future and present utility values. Although research in the area has identified several psychological biases in the decision making process, most research explores one bias at a time, given the computational complexity of considering more than one, among other reasons. This program of research proposes to create a numerical platform for exploring the implications of how countervailing biases may interact to create unexpected outcomes when two or more biases are present at the same time. It will use life-time savings decisions as a theoretical domain since both theory and empirical studies are well-developed. Our program of research involves four main stages: 1) analyzing the "Individual Utility Function" model in behavioral economics, 2) developing a simulation platform to explore strategies to maximize lifetime utility incorporating four biases widely-explored in the behavioral economics literature, 3) using the platform to explore main interactions among the four biases, and 4) reflect on the process and results to contribute to the field of behavioral economics. In this paper, we introduce these four steps, and also discuss initial progress in stages 1 and 2.*

1. Introduction.

After the advent of computers, research in economic dynamics has transformed dramatically. Mathematics alongside with computers could help experts to run more complex algorithms and optimization techniques. Using these new tools, it is possible to tackle problems which used to be unsolvable in the past (Stachurski, 2009). Without computers, empirical researchers used to add dynamic and stochastic elements as afterthoughts to raw outcomes of static, deterministic economic models. However, with new simulations, it is now possible to explore and test powerful theories about rational economic agents operating through time in stochastic environments (Stokey, 1989). There are many different methods for theoretical models, in which we see complicated stochastic processes to investigate economics behaviors. We can track the history these theoretical developments over time by reviewing the literature. Ramsey (1928) and Hotelling (1931) firstly established the economic applications of the calculus of variations. Later on, the contingent-claim view of economic equilibria was introduced by Arrow (1953) and Debreu (1959). The contribution of Bellman (1957) and Blackwell (1965) was the theory of dynamic programming. In 1989, Stokey studied the Recursive Methods in Economic Dynamics.

The ultimate goal in this program of research is to provide a numerical platform for simulation-based exploration of theoretical ideas that are grounded in the modern economic dynamics. Our approach contributes to the literature on behavioral economics because it provides researchers with graphical representations that will potentially help the exploration of more complex models, and also through these visualizations reach a wider audience of researchers that may apply behavioral economics models to domains such as natural resource management. In addition, these simulations can support both deterministic and stochastic dynamic economic problems, problems facing certainty or uncertainty, and finally problems in either finite horizon or infinite horizon. Combined with the System Dynamics concepts such as accumulation, feedback loops, delay, and graphs over time, economic dynamics would carry new hypotheses which can be tested in a computational experimental environment.

To explain how System Dynamics models can help us to better understand a behavioral economics problem, we started with a well-established macroeconomics problem. Maximizing the lifetime utility of an individual's consumption behavior is an example of many other concepts in economics that can be studied this way. This example also serves to illustrate the kinds of substantive economic questions are drawn from the much longer list of applications to be treated in detail in later researches.

The rest of the paper is organized in three additional sections. Section 2 describes our theoretical motivation. Section 3 reports on our current progress in the development of our numerical simulation platform. Finally, section 4 briefly describes the implications of our current results.

2. A Multi-Step Research Program

2.1 Step one: Analyzing the "Individual Welfare Function" Model in Behavioral Economics.

We will use one of the better studied areas within behavioral economics, the problem of deciding how much to save for the future. The model that is presented here starts from one of its simplest version. To simplify the model we need some assumptions to make.

People live T periods, and retire in period τ. In each period $t < \tau$, they get some noisy income realization and earn some noisy investment income. They then decide how much to consume and how much to invest. In periods τ to T, the only source of income is investment income. One modeling decision is whether to give them a choice between a liquid and an illiquid asset to invest in. Another modeling decision is whether to give them a risky asset and a safe asset. Individuals may deal with a safe liquid asset, like government bonds, or a risky illiquid asset, like housing.

Current assumptions are as follow: 1) τ=T, which means individual will have income in all time periods. 2) income increases with a steady growth rate, 3) the investment interest rate (r) is a constant, 4) we started with only one type of asset.

Fully Rational Model

The model will run for T+1 number of time periods. Each individual start working at t=0 and dies at=T. The purpose of the model is maximizing the *Lifetime Utility (U).* Lifetime utility is the extent of happiness the household derives from its consumption. *Instantaneous Utility (u)* is how much utility the household gets in a given period by its *consumption (c_t)* in that period. The *discount factor (δ)* is how much less people care about the future compared to the present. The discount factor is a value between zero and one ($0< \delta<1$). *Coefficient of relative risk aversion (ρ)* is how much someone dislikes fluctuations in consumption. *Labor income (Y_t)* is what the person receives in each period and it excludes the *interest income (r)*. In this model, we assumed that the income grows with the constant rate of *G*.

$$Y_t = GY_{t+1}$$

Utility increases as the individual consumes more. However, consuming the first unit of goods yields more utility than the second unit. In general, by consuming more, the utility increases at a decreasing rate. There are numbers of utility functions that can reflect this effect. The following equation is one of the popular utility functions which is also known as iso-elastic utility function (or constant elasticity).

$$U(\{c_t\}) = \sum_{t=0}^{T} \delta^t u(c_t) = \sum_{t=0}^{T} \delta^t \frac{c_t^{1-\rho}}{1-\rho}$$

With this utility function, we will have:

$$u'(c) = c^{-\rho}$$

The Constraint is the income process. In each time period, we can calculate the corresponding wealth (W_t) by using:

$$Wt = (1+r)\,(W_{t-1} + Y_{t-1} - c_{t-1})$$

Optimal solution

The optimal dynamic programming solution has people saving to finance consumption in retirement and as a precaution against negative income shocks. You backwards induct a decision rule about how much to consume each period as a result of inherited wealth and any state variables about the income process. For simplicity purposes we did not consider state variables in income.

The solution for this optimization method is the "Euler Equation":

$$u'(c_t) = \delta(1+r)Gu'(c_t+1)$$

$$c_t^{-\rho} = G\delta(1+r)c_{t+1}^{-\rho}$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\rho} = G(\delta(1+r))$$

For finite horizon model, you would backwards induct to get the solution to how much to consume any given wealth state. Individuals will not save for the time after T+1. So, in the last period, the individual will consume everything that is left[1].

2.2 Step Two: Developing a Simulation Platform.

The Simulation model should exactly captures important aspects of economic theory in the domain of lifetime savings. Based on the theory and current assumptions, we expect from the model that an individual save more in the early and middle ages of his lifetime and he will consume his savings at the end.

[1] For an infinite horizon model, one would use numerical dynamic programming methods to compute the fixed point of the Bellman equation.

$$V(W_t) = max\, u(c_t) + \delta V(W_{t+1} + Y_{t+1} - c_{t+1})$$

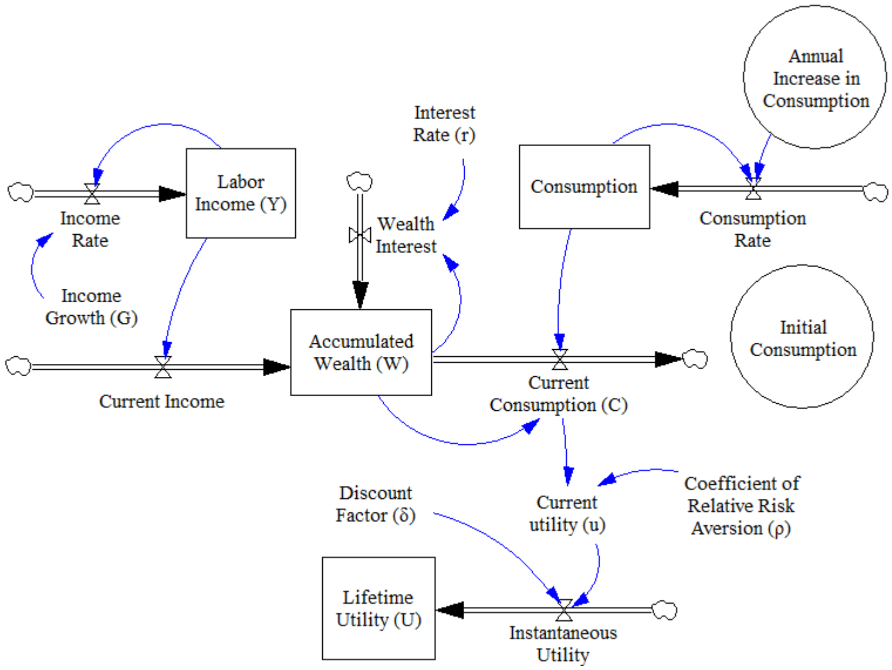

Figure 1. The SD translation of "Individual Welfare Function" Model

Each equation in this model has an equivalent in the behavioral economic model which was discussed in the first step. But how do we know whether our simulation shows the exact same results that we captured from the first step? In the first step, we claimed that the optimal consumption behavior should follow the Euler Equation. To build confidence in our model, we will optimize it subject to the two exogenous variables that are used in the Euler equation, the *Initial Consumption* and the *Annual Increase in Consumption (AIC)*. If the outcomes optimization equals the values that the Euler equation suggests, our model is correct. The other check can be the behavior of the *Accumulated Wealth (W)*. This variable should create a complete or a skewed bell-shaped graph over time. In addition, it should get the minimum value at the final step, as the person cannot consume after his death. The followings are the outcomes of our models which perfectly matches our validation scenarios:

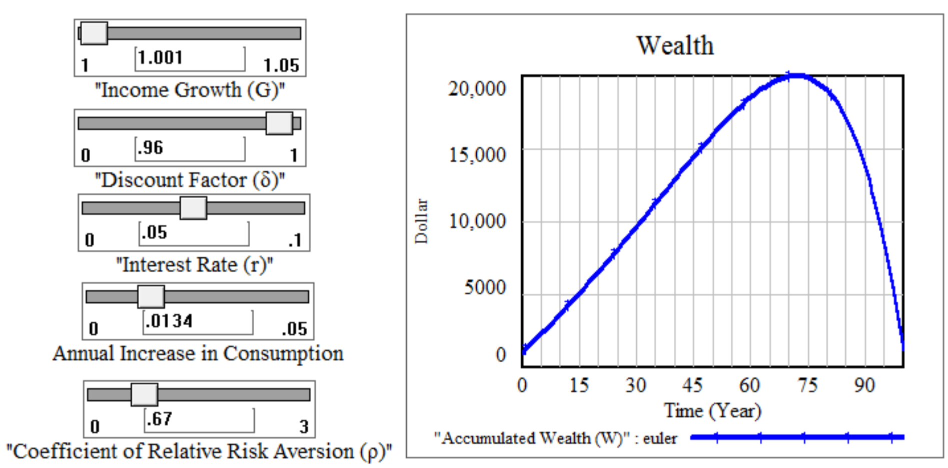

Figure 2. The Behavior of Accumulated Wealth over time based on variable shown on the left.

In the model, the individual will have an income even in the last time step. So, wealth in the last time step is not exactly zero. It equals to the income in that time step.

In this model, the equation for *Consumption Rate (CR)* is:

$$Consumtion\ Rate = Consumption * Annual\ Increase\ in\ Consumption$$

Using the mathematical equations behind this formulation, we will have:

$$c_{t+1} - c_t = c_t * Annual\ Increase\ in\ Consumption$$
$$c_{t+1} = c_t * (Annual\ Increase\ in\ Consumption + 1)$$
$$\frac{c_{t+1}}{c_t} = (Annual\ Increase\ in\ Consumption + 1)$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\rho}_{simulation} = (1 + Annual\ Increase\ in\ Consumption)^{\rho}$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\rho}_{Simulation} = (1 + 0.0134)^{0.67} = 1.009$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\rho}_{Euler} = G\big(\delta(1 + r)\big) = (1.001) * (0.96) * (1 + 0.05) = 1.009$$

Since the value from the simulation equal the value derived from the Euler equation, we can claim that the simulation can mirror the results of the first step.

2.3 Further research

Step Three: Use the Platform to Explore All Four Biases. This step replicates research from Step one, but finds a way to analyze and present interactions between all four biases. Behavioral biases: we want to have consumers form their solutions in the face of four counter-vailing biases:

1) **Present-biased preferences** (a.k.a. hyperbolic discounting): the IWF either doesn't have a preference over the T periods or has a fixed exponential discount rate (so it might value a util in period 30 as only δ^{30} as much as a util in period 1, where $0 < \delta < 1$). With present-biasedness, an agent discounts period 30 as $\beta\delta^{30}$, where all future periods are discounted by a hyperbolic discounting factor $0 < \beta \leq 1$. This creates time inconsistency, where people want to save for the future when they are comparing two future periods, but generally want to consume today when comparing today to a future period.

   Effect on savings: shift savings to the illiquid asset
   decreases savings

2) **Overconfidence:** agents think their actual market return will be higher than it actually is.

   Effect on savings: increases savings

3) **Myopic loss aversion:** agents face a jolt of disutility based on how much their savings go down between periods t and t+1.

Effect on savings: shift savings to the safe asset
decrease saving

4) **Projection bias:** agents expect their future needs and incomes to be similar to current needs

Effect on savings: Shift savings to the illiquid asset
Decreases precautionary saving

Each of these biases is defined by a single parameter. There is thus a vector of four behavioral parameters, θ

For any given vector θ, an individual would have a different savings rule mapping the time period t, current wealth level, and current income level into consumption and investment decisions. That rule then produces a consumption stream {c}, which can be plugged into the Individual Wealth Function to get a total value.

We want the simulation to be able to map the space of θ. Under standard behavioral economic modeling, increasing any bias always makes people worse off. If multiple biases push in different directions, then there should be regions where increasing one of the biases makes people better off. Then, we will look for local maxima in the space of θ; at such points, a policy to reduce the impact of a bias would reduce their welfare. Using this method will give us a policy tool (such as illiquid savings accounts) and an opportunity to look at what parameter regions tend to make them a good idea.

Step Four: Theoretical Elaboration. Steps one and two are built on a numerical platform that accepts all of the assumptions of the standard economic model as well as empirical research on behavioral biases. When viewed from a SD perspective, these frames are low on implicit feedback effects. But our numerical platform has been cast exactly in the form of a system dynamics model, so it will cry out for the addition of dynamic hypotheses that rely on feedback mapping. The contribution here will be an elaboration of existing theory in behavioral economics, leading to proposing empirical methods for testing these new dynamic hypotheses.

3. Implications of this Research Program

The purpose of our proposed research program is to develop a numerical platform that uses simulation to facilitate experimentation of complex models in Behavioral Economics. Our final vision involves the development of a platform that can be used to test interactions between four well-researched biases in human decision making.

Although our research is still in early stages, we believe that the results are promising. We have developed an initial prototype that proves that it is possible to model and optimize the basic assumptions in behavioral economics with regards to wealth, consumption, income and lifetime utility. Our results suggest that it is possible to develop a more tightly coupling of system dynamics simulation with empirical work in behavioral economics, adding both theoretical insight, suggesting new pathways for research, and opening new domains for future research.

A great variety of economic and other decision-making problems are quite naturally cast in a recursive framework. Many of these problems, especially those regarding our consumption decisions on natural resources, are particularly relevant in the current discussions on sustainability and development.

4. References

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